Computer-based Mathematics and Physics for Gifted Students[1]

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January 7, 1994

[1]Research supported by the National Science Foundation Grants MDR-90-50008, MDR-85-50596, and MDR-87-51523 at Stanford University. Request for reprints should be sent to Raymond Ravaglia at CSLI, Ventura Hall, Stanford University, Stanford, CA 94305-4115. *Note: This article appeared in the Winter 1995 (Vol 39, No I) Issue of* Gifted Child Quarterly.

Abstract

Computer-based education makes it possible for gifted and talented middle school and early high school students to complete advanced courses in mathematics and physics several years before they would normally do so. Since the fall of 1990 three such groups of students at the Education Program for Gifted Youth at Stanford University have taken courses at the advanced placement level and have done exceedingly well. This report details these results.

1 Introduction

A recent report by the U. S. Department of Education (1993) laments the poor performance of the best American students compared with students from other developed countries. The top 1% of American students ranked 12th and 13th out of 13 in the categories of secondary school and college-level mathematics. This report goes on to call for programs that would allow gifted students to begin their studies at an earlier age and others that would offer more advanced courses to younger students.

What we describe in this paper is a relatively new program, the Education Program for Gifted Youth (EPGY) at Stanford University, which incorporates both of these features. EPGY aims to provide year-round, accelerated instruction in mathematics and physics to gifted students via a computer-based curriculum, thereby allowing these students to complete the standard secondary school mathematics courses and progress into university-level courses before leaving high school. Because the program is computer-based it places no limits on the number of participants at a site and indeed does not restrict participation to students living in any particular location.

EPGY students run the multi-media courseware at home or in school on standard IBM-compatible personal computers. This software, unlike traditional computer-based educational programs, is intended to stand alone, not merely to supplement a regular class. The computer presents lectures using digitized sound and graphics, in essentially the same way that a human instructor would. These lectures are followed by on-line exercises that gauge the student's understanding as would a teacher in front of a classroom, but in greater detail. Students' only direct contact with instructors is electronic, via electronic mail and telephone contact with centrally located project staff.

We have used this course model over the last three years to teach Advanced Placement Calculus and Physics to gifted middle school and early high school students. The remainder of this article documents our experience and discusses the role that programs like EPGY might fulfill in the future.

2 The Evolution of EPGY

EPGY evolved from two distinct strands of research conducted at the Institute for Mathematical Studies in the Social Sciences IMSSS at Stanford University, under the direction of Professor Patrick Suppes. One strand involved computer-based education; the other involved the education of gifted children.

Computer-based projects at IMSSS have included instruction in mathematics from elementary arithmetic to college level logic and set theory, as well as courses in a number of languages, including Russian and Armenian. These projects are reviewed extensively in Suppes (1981). These early studies demonstrated that students can benefit significantly from on-line instruction, whether the computer merely provides drills on concepts learned in the conventional classroom or provides a complete tutorial format including both exposition and practice. Moreover they showed that among students of all levels and abilities there are significant differences in the rates at which students will move through the courses, demonstrating the importance of an adaptive curriculum sensitive to individual differences. A detailed examination of these points for both students in the secondary school and university levels are given in Larsen, Markosian and Suppes (1978) and Suppes, Fletcher and Zanotti (1976).

IMSSS also conducted investigations of gifted education in mathematics concurrent with, and sometimes coincident with, these early experiments in computer-assisted instruction. In the first such study, conducted in 1955, students in the ninth grade were invited into an introductory logic course at Stanford. These accelerated high school students performed at the same level as the college students. A later, more ambitious project involved a longitudinal investigation of an accelerated program in mathematics for gifted elementary students conducted from 1963 to 1967 (Suppes, 1966, Suppes & Hansen, 1965, Suppes & Ihrke, 1967, 1968). The program began with a group of first graders who worked at their own pace through mathematics textbooks (Suppes, 1963, 1964, 1966), attended classes, and worked on a variety of supplemental materials, some of it computerized. Students were exposed to topics and concepts not normally taught at the elementary levels, such as logic and geometric constructions, in addition to abundant drill in the traditional curriculum.

Several striking findings emerged from this investigation. First, students on the whole worked very quickly through the materials presented. Nevertheless, they varied widely in the rate at which they worked through the self-paced materials—by the end of the fourth year, two textbook years separated the fastest from the slowest participant. Students also differed widely in the proportion of errors they committed, though there was surprisingly little correlation between speed of progress and number of errors committed. Especially notable, particularly given the disparity among student work rates, was the uniformly high scores they achieved on standardized tests keyed to the material they had completed. These findings highlight individual differences in learning rates even among the gifted, and consequently stress the need for self-paced materials to allow each such student to perform at his or her maximum achievement potential.

The present Education Program for Gifted Youth grew out of this research in computer-assisted instruction and gifted education. Development of what later became the EPGY courses began in 1985 at IMSSS, funded by a grant from the National Science Foundation, as proof of concept demonstration that a first-year college calculus course could be entirely computer-based. The original motivation to design an online calculus course was the fact that fewer than 25% of the high schools in this country currently offer calculus; the idea was to make this course available at schools having interested students but either no one qualified to teach them or no money to justify having a small class for just a few students. Our experiences while selecting a test site for the calculus course, as well as several fruitful conversations with Julian Stanley, founder of the Center for the Advancement of Academically Talented Youth (CTY) at Johns Hopkins University, suggested to us that our program might prove especially valuable for very gifted students in middle schools, schools that are even less likely to have resources available for teaching calculus to small numbers of students. Hence we decided to test our program on the groups of talented younger students who are the subjects of this report.

3 The EPGY Advanced Placement Courses

Three groups of students have participated in EPGY at the advanced placement level since the fall of 1990. The first group, designated Calc91, consisted of students in seventh through tenth grades who took the Calculus AB course in 1990–91. The second group, Calc92, consisted of students in the seventh and eighth grade who took both the Calculus AB and the Calculus BC courses in 1991–92. The third group, Phys93, consisted of students in 8th through twelfth grades from Calc91 and Calc92 who had successfully completed the EPGY calculus courses and went on to take the Physics C: Mechanics course in 1992-93.

The Calculus AB, Calculus BC and Physics C: Mechanics courses teach material covered by the Advanced Placement tests having the corresponding titles. Calculus AB covers the first two quarters of college calculus; Calculus BC covers the entire first year of college calculus, including a more thorough treatment of the topics in Calculus AB; and Physics C: Mechanics covers a standard semester of college physics with a calculus prerequisite.

3.1 Selection of Students

Students in Calc91, (Calculus AB 1990–1991), were drawn from the pool of applicants to the summer session of the Academically Talented Youth Program (ATYP at Foothill Community College, near Stanford University, which draws students from San Mateo County to the north and Santa Clara County to the south of Stanford.[1] A group of 92 potential students was selected from a total pool of 300 according to their PSAT-Q scores by age. For this group we set cutoff scores of 500 for a student entering grade 6 or 7, 550 for a student entering grade 8, 600 for a student entering grade 9, and 650 for a student entering grade 10. From this group of students, 30 enrolled in a summer precalculus class we offered at Foothill College, and of these 17 (9 boys and 8 girls) passed the course with a level of understanding indicating a good chance of success in calculus. Of these 17 students, 15 (7 boys and 8 girls) accepted our invitation to participate and formally enrolled in the calculus course. Thirteen of these 15 students completed the course. Their PSAT-Q scores are shown in Table 1.

During the summer of 1991 we drew from the next year's pool of applicants to the ATYP program a group of participants for a newly designed series of courses intended to take students though the standard secondary school mathematics curriculum and prepare them for calculus. In order to focus on middle school students, we restricted our selection procedure to those who would be starting the sixth, seventh or eighth grade in the autumn of 1991. We found that four individuals selected for this group (three boys and one girl) seemed already prepared for calculus, judging from their remarkably high PSAT-Q scores (see Table 1) and prior mathematical experience, and

[1]We are grateful to Dr. Janice Carr of ATYP for her generous cooperation.

Group	S	Grade	Sex	PSAT-Q
Calc91	1	7	F	660
	2	8	F	690
	3	8	F	680
	4	8	M	680
	5	8	M	560
	6	9	F	520*
	7	9	F	700
	8	9	M	660
	9	9	M	740*
	10	9	M	650
	11	10	F	630
	12	10	F	620
	13	10	M	660
Calc92	1	7	M	800
	2	8	M	730
	3	8	F	730
	4	8	M	800

* PSAT-Q score is from 1988 (6th or 7th grade). Others are 1989 or early 1990.

Table 1: Calc91 and Calc92 Selection Information

it is these students who constituted our second group of calculus students (Calc92). These students, unlike those in Calc91, did not take a summer preparatory course; thus our only selection criterion was PSAT-Q scores. All four accepted our invitation. These students took the same Calculus AB course offered earlier to Calc91 followed by a newly developed Calculus BC course. Because we wished to concentrate on middle-school students we did not offer the new Calculus BC to any of the participants in Calc91.

All students from Calc91 and Calc92 who successfully completed their respective calculus courses were invited to participate in Phys93 (Physics C: Mechanics, 1992–1993). All of those from Calc92 chose to participate, but only six (3 boys and 3 girls) of the 13 from Calc91 accepted the invitation. The seven who did not participate cited heavy course loads or scheduling conflicts at their schools as the primary reason for not accepting.

Student	Grade	Sex	Prior AP Exam and Score
1	8	M	BC–5
2	9	M	BC–5
3	9	F	BC–5
4	9	M	BC–5
5	9	F	AB–5
6	10	M	AB–5
7	10	M	AB–4
8	11	M	AB–4
9	11	F	AB–4
10	12	F	AB–4

Table 2: Phys93 Selection Information

3.2 Student Background

It should be emphasized that in selecting the students for the Calc91 and Calc92 groups, we did not base our selection on courses already taken, but only on scores received by the students on the quantitative section of the PSAT. For Calc91, since we planned to give them an intensive summer course in precalculus, we used a lower selection criteria than we did for students in Calc92, who were not given any preparation before starting the course. The scores of the students are listed above in Tables 1 and 2.

Of the students in Calc91, all but the student entering seventh grade had completed a course in first year algebra, and four of them had completed courses in both geometry and the second year of algebra. The precalculus course that these students took over the summer assumed that students had completed a course in beginning algebra, but assumed nothing else.

Of the students in Calc92, none of them had received more than one year of formal instruction in algebra. Two of them had received informal instruction in other topics in mathematics from private tutors. These students did not take a course in precalculus, but instead were given a precalculus text book to use as a reference source (Lial, Miller, 1988). Only one of these students claimed to make any use of this book.

4 Description of Courses

4.1 Overview

Preparation. Only those students in Calc91, who it will be recalled attended a summer course in precalculus, received formal preparation in advance of their computer work. Students in Calc92 were tested for specific knowledge before beginning their computer work and were given material to review as necessary on their own. Although all students in Phys93 had the requisite mathematical background for the physics course, having completed calculus with us, only two of them had taken any physics before. We addressed this general weakness in students' physics background by giving them a conceptual physics text book to read on their own before beginning the formal course work (Kirkpatrick, 1992).

Hardware. During school years 1990–1991 and 1991–92 students ran the calculus courses in their schools on UNIX workstations. These students were visited by a teaching assistant at their schools twice a week in the case of Calc91 and once a week in the case of Calc92. Calc92 also had phone and fax support available during certain hours of the week.

During the 1992–93 school year, the Phys93 students ran the physics course at home on personal computers under the Windows operating system. They were in contact with Stanford instructors by phone and electronic mail. In addition to this they could attend discussion sections at Stanford, which were offered once every three weeks.

4.2 Course Design

All EPGY courses use computer-based methods as the primary vehicle of instruction, although some assistance from a tutor who can discuss or correct off-line work, grade tests and certify performance in the course is required. The following course components are on-line: a complete, interactive, multimedia exposition of the curriculum material involving digitized sound and graphics, an interactive problem-solving environment, mastery quizzes, problem sets, and databases of off-line problems. All the courses incorporate the Maple computer algebra system (Char, 1988) as the internal computational engine for doing algebra and calculus operations. Along with the engine for symbolic computation, the courseware incorporates a curriculum driver, authoring system, interactive interface for calculus problem solving, and graphing facilities.

For each topic within a course, the computer first presents an interactive lesson of about an hour's duration. In the Calculus AB course this lesson typically consists of 10-20 minutes of interactive textual exposition, including 20–40 nontrivial interactions with the computer (short answer, multiple choice, working with a graph, etc.). In the Calculus BC and Physics courses, however, we wanted to preserve more closely the informal discursive character of spoken mathematics or physics. Instead of interactive textual exposition a lesson in these latter courses involves a brief lecture (usually under 5 minutes) consisting of digitized sound recordings along with handwriting or formatted text and graphics that appeared on the computer screen in real time, more closely resembling what a teacher would write on a chalkboard while lecturing. These lectures alternate with a set of interactive exercises, such as a set of questions about the preceding lecture, interactive exposition in which the student is led through a detailed argument step by step, or a derivation in which the student is asked to prove a mathematical fact. One lesson generally comprises three or four lecture-exercise units.

It should be stressed that the amount of material presented to a given student varies according to that student's rate of mastery of the material. After the computer has presented the initial instructional material, students are given exercises testing their understanding of that material. Depending on their responses students either proceed to the next subject or receive focused remediation until they understand it. This technique of quick presentation followed by diagnostic tests and additional instruction tailored to the students weaknesses has been demonstrated to be a crucial aspect of projects which are geared for allowing students to accelerate rapidly (Stanley, 1991).

Students do off-line work in addition to their on-line lessons. In fact, for each hour they completed on-line, participants spent as much as 45 minutes doing additional off-line homework—reading and exercises in traditional texts (Anton, 1988) and (Tipler, 1991)—both to give them more practice with the material and to help them prepare for off-line examinations.

At the end of a lesson topic, the computer presents another quiz on material covered since the previous quiz. Additional remediation is provided to those students needing it. For the students reported on here, off-line examinations were administered at the end of every chapter (corresponding to chapters in the text). The complexity of the exams varied from simple 60 minute in-class exams to take-home exams to be turned in a week later.

It should be evident that the tutorial role of the computer in EPGY courses is considerably more central than the role technology frequently plays in mathematics or science education. The computer in our program is no mere computational aid, electronic textbook, or drill assistant. Nor is it an enrichment supplement like a mathematics or physics lab or optional aspect of another instructional modality. Rather we sought and continue to seek to exploit the technology as fully as possible to produce stand-alone courses that capture and maintain students' interest while efficiently teaching substantial academic content.

From the students' viewpoint, the courses are managed by the computer, but in important ways students must manage themselves more than they would in a classroom or live tutorial setting. Students must find their own pace, take action when difficulties arise, and keep up with course requirements. Subject to several examinations or milestones during each semester of the full-year course, students have flexibility in pace, workload, and help from human tutors. In particular, they are free to spend as little or as much time as they need to master material. We recognize that the self-regulatory aspect of the EPGY courses may prove beneficial for some students but not for others. Our past experience with self-paced courses, including twenty years of running a self-paced course in logic at Stanford has taught us that monitoring student progress and periodic personal contact are necessary to motivate students who might otherwise tend to procrastinate. These points are discussed more completely in Macken, van den Heuvel, Suppes, and Suppes (1976) and Suppes (1981).

4.2.1 Additional Remarks on the Physics Course

In addition to the components discussed above, the physics courses we offer include laboratory work. Because laboratory work is an important component of any physics course, we felt that it was essential to make labs available to these students. We have also tried to avoid relying on simulations, preferring to give these students actual hands-on experience. This is particularly important in the case of gifted students who are very advanced in mathematics. It is natural for them to forget that physics is about the behavior of the real world. As such making them do physical experiments helps to disabuse them of this notion.

Starting with the Fall of 1993, EPGY has begun to offer a course in Physics C: Electricity and Magnetism. In conjunction with this course we have offered a laboratory section in which students do experiments at home. The labs in this section come from the text "Zap! A Hands-on Introduction to Electricity and Magnetism" developed by the Zap! project, a joint effort involving physicists at MIT and CalTech (King, Morringson, Morringson, and Pine 1992). The Zap! course is based on a series of experiments which utilize roughly $60.00 worth of common tools together with $40.00 worth of electronics hardware. The kit makes it possible for students to do a wide range of interesting experiments ranging from the verification of Ampere's law to Hertz's experiment showing the propagation of electro-magnetic waves, and concluding in the measurement of the speed of light.

We will discuss our experience conducting these laboratory sections remotely in a future publication.

5 Results

Tables 3, 4 and 5 present EPGY students' scores on Advanced Placement (AP) exams from test years 1991–93. AP scores can range from 1 (lowest) to 5 (highest). The most striking aspect of students' scores is that they are generally quite high. 92% of those having taken Calculus AB, 100% of those having taken Calculus BC, and 88% of those having taken Physics C received scores of 4 or 5, with the majority of these receiving the highest score possible. Nationally one would expect only 45%, 57% and 47% of students to score 4 or 5 on these exams, respectively. It should be further emphasized that these numbers are for all students, whereas the majority of the EPGY students have been in 9th grade or below when they have taken exams. We are confident that the ratio of average score divided by student age for our mechanics class, Phys93, is among the best in the world. Not only have our young participants successfully completed substantial college-level material as a result of their EPGY course work, they have done exceedingly well.

Another notable feature of our results is the lack of sex differences we found. The numbers of girls and boys among those first invited to participate in EPGY and among those who finished the courses were nearly equal. Boys and girls who completed the courses produced nearly equally outstanding

S	Grade	Sex	AP Score
1	7	F	5
2	8	F	5
3	8	F	5
4	8	M	5
5	8	M	4
6	9	F	5
7	9	F	5
8	9	M	4
9	9	M	4
10	9	M	3
11	10	F	4
12	10	F	4
13	10	M	4

Table 3: 1990-91 Calculus AB Exam Results

S	Grade	Sex	AP Score
1	7	M	5
2	8	M	5
3	8	F	5
4	8	M	5

Table 4: 1991-92 Calculus BC Exam Results

outcomes. These results are a bit of a surprise in light of the more usual findings that gifted and talented programs, especially in mathematics and science, tend to include many more boys than girls (Stanley, 1991 & Crombie, Bouffard-Bouchard & Schneider, 1992) and that boys generally tend to find computer-based activities more attractive than do girls (Hess & Miura, 1985 & Kiesler, Sproull & Eccles, 1983). Perhaps selecting students who are fairly young (i. e. in middle school rather than high school), reduces the cumulative effects of sex-biased socialization. Another interesting possibility, is that when one selects for very talented students using tests that are primarily ability tests, one eliminates the gender-biased perceptions of teachers and

Student	Grade	Sex	Prior Calculus Exam & Score	Physics C Mechanics Score
1	8	M	BC–5	5
2	9	M	BC–5	4
3	9	F	BC–5	5
4	9	M	BC–5	3
5	9	F	AB–5	5
6	10	M	AB–5	5
7	10	M	AB–4	4
8	11	M	AB–4	4
9	11	F	AB–4	4
10	12	F	AB–4	NA

Table 5: 1992-93 Physics C: Mechanics Exam Results

parents, who tend to refer twice as many male students as female students to us for testing (and indeed, tend to refer more boys than girls to gifted and talented programs nationally (Stanley, 1991)), whereas the ratio of boys to girls who meet our objective test criteria is about 1:1.

Our low attrition rates deserve special mention. Of the 15 students who started Calculus AB in Calc91, 13 (87%) completed the course. Of the two who did not, one student cited the inconvenience of walking from her high school to the local middle school in order to take the class as the reason she dropped. The other student who dropped had been doing quite well in the course, but wanted to have more time to devote to his track team. The only other student we have had who started a course but did not take the AP exam was a young woman in Phys93 who finished the course with a grade of A, but did not take the AP Exam because she was going to CalTech the following year and they do not accept AP Exams.

6 Advantages of EPGY

The potential advantages of home computer-based instruction for the gifted are numerous, from both the point of view of the students and of school administrators controlling limited resources. Some of these advantages have already been mentioned elsewhere e. g., Sowell (1993). We now discuss some of the specific benefits of EPGY in more detail here.

First, students in EPGY are allowed to work at their own pace and in their own home. The home-based feature is particularly significant for younger students because it eliminates the need to travel or rearrange their schedules to conform to the course offerings of local colleges or universities. Traveling and scheduling are burdensome and time consuming for everyone, but they pose particularly difficult hurdles for students who are too young to drive or who live far away from centers offering gifted and talented programs. Possibly more important is the fact that students who run these computer-based courses need not bear the burden of awkwardness that would be imposed were they to attend classes full of students significantly older than they are. Another advantage to the EPGY program is that unlike many programs for the gifted and talented, which tend to run only during the summer, or only one day per week, the EPGY curriculum is available to students day-to-day on a year-round basis and new students may enroll at any time throughout the year. A program such as EPGY can make it possible for qualified students to take advanced courses years before they would normally be able to, even if they were regular participants in summer gifted programs. And furthermore it means that they do not need to artificially divide their education, taking advanced courses during the summer and standard ones during the year.

There are advantages to EPGY from the schools' perspective as well. Few middle schools are able to offer courses at a level beyond Beginning Algebra, nor high schools courses beyond Calculus. EPGY makes it possible for these schools to address the needs of their most advanced students by providing them with courses commensurate to their ability level, without the added expense of hiring a teacher for the specific purpose of teaching these students. By having students participate in a program such as EPGY, in which students run courses at home, schools are able to keep their students in their normal school setting and address their educational needs without depleting school budgets.

There is one more point which needs to be made. It has been fairly pointed out on several occasions, and we have in fact been aware from the start, that students of this caliber are able to learn what is presented to them, no matter how it is presented to them, and that the success we have had should be attributed more to the students than to the software. This is not, however, a criticism of EPGY, but a point in its favor. The fact of the matter is that students of this ability are able to learn a wide body of advanced material, provided they are in a structured setting which can compete with their otherwise busy schedules and permit them to do the work. This is the key to our program. It provides just such a structured environment and it does so in a way that is convenient to all involved. Because students are able to run the course in their own time it does not force them to choose between the course and something else, but in essence lets them have their cake and eat it too. Moreover, while it is true that our students have been very gifted, they are by no means very rare. While there are no reliable national norms for scores on the quantitative section of the PSAT for students of this age, looking at the data published by Johns Hopkins Center for Talented Youth (Center for Talented Youth, 1993) we suspect that our students represent the top 1% of the population. Whether other students in the population would have similar successes in our courses, or in a modified version of these courses is something that we will have to determine with future research.

The EPGY program provides a way to deliver systematically an advanced body of instructional material to a very select and geographically diverse group of students. Our goal is to make it possible for the very best students in mathematics and physics to obtain, while still in secondary school, an education equal in content to those given in the top high schools in the world, including those specializing in mathematics and science.

7 Future of EPGY

So far, we have discussed only a small portion of our work in this article. Following the success of the Calc91 group on the 1991 Calculus AB Exam, and the realization that students of that age are capable of success in the calculus, provided that they are given a thorough mathematical background, we went immediately to work on developing a sequence of courses designed to cover in a concise way the bulk of secondary-school mathematics.

Towards this end we have developed a course sequence which takes students from Beginning Algebra to the point were they are ready to begin the Calculus. Our goal with this sequence is to make it routine for advanced students to start Beginning Algebra in the sixth grade, and to complete Intermediate Algebra and Precalculus by the end of the seventh grade. In this way these students will be able to take a full year of calculus in eighth grade.

The first group of students started this sequence early in 1992 and we expect to have about 15 students in seventh to ninth grade taking Advanced Placement Exams in May 1994. There are currently 150 students enrolled in the EPGY secondary school course sequence and we expect this number to double by the end of the 1993–94 academic year.

We have also developed a wide variety of support techniques using both synchronous communications, e. g. phone calls and teleconferencing, and asynchronous communications, e. g. electronic mail, to make it possible for students to participate in the courses in situations where it is not feasible for them physically come to Stanford University. These facilities and our experience with them will be detailed elsewhere.

The 1993–94 academic year also marks the first time that we have offered the Physics C: Electricity and Magnetism course. We had 12 complete this course, 9 of the students from Phys93 together with 3 new students, including one who was in Calc91 but who had not taken courses with us since then. All 12 of these students took the Physics C: Electricity and Magnetism Advanced Placement Exam in May of 1994.

As it stands the Stanford Education Program for Gifted Youth currently offers enough courses for a student to compile 29 quarter units of college credit. We hope eventually to add mathematics courses in Multivariate Calculus, Linear Algebra, Differential Equations, as well as Physics courses in Optics, Thermodynamics and Modern Physics.

The College Board's Advanced Placement program has made it possible for a large number of high school students to take introductory college courses, but it has done nothing to provide courses that go beyond this introductory level. Nor would it be feasible at the present for them to do so, for it is not possible for schools to devote the teaching resources necessary to regularly offer courses beyond the advanced placement level. We hope that EPGY will be able to do so by offering a large number of college-level courses to the widely dispersed group of very gifted students in this country.

8 Afterthoughts

There is a current tendency among some middle-school educators to recommend that middle-school students not be accelerated in mathematics. These educators claim that the mathematics curriculum is being widened and that students are now being taught much more than ordinary mathematics. These educators add that if one were to accelerate students, they would miss these new parts of the curriculum, and that the end result would be that the education they received, although accelerated, would be worse than that of their unaccelerated peers.

The problem with this trend is that what is meant by deepening the subject matter more often that not amounts to making mathematics more interesting by making it less mathematical. While this may be an effective way to teach mathematics to some segments of the population, our feeling is that students who are capable of doing so, should move quickly to the point where they can actually do serious mathematics and apply it in their sciences courses. Acceleration for our students has enabled them to study serious material at an earlier age and prevented them from wasting time doing courses like physics once without calculus and to later repeat the same material with calculus.

Our experience with these very able students suggests that they can make tremendous gains if they are allowed to move at their own pace through material which they find extremely easy. This is not to say that such students go through the rest of their mathematics education at a uniformly rapid pace. Rather these students tend to slow down somewhat in their pace as the material becomes more difficult, with most of the students spending more than two quarters to complete BC Calculus, slightly faster than the average college student.

We can only conclude from our experience that for these students, acceleration is appropriate, provided the student is allowed to move at his or her own pace, and is required to demonstrate mastery of the material throughout. To claim that their education is improved by not letting them learn basic material at a faster pace, if they are clearly able to do so, simply does not ring true.

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